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Title : Bishop-Phelps theorem and its latest developments

Abstract : The celebrated Bishop-Phelps theorem [1], that is, “the set of norm attaining linear functionals on a Banach space X is dense in its dual space X^* ” appeared in 1961. Bollobás [2] sharpened in 1970 the Bishop-Phelps theorem by dealing simultaneously with norm attaining linear functionals and their norming points, the so-called the Bishop-Phelps-Bollobás theorem, which is stated as follows: Let X be a Banach space and $0 < \epsilon < 1$. Given $x \in S_X$ and $x^* \in S_{X^*}$ with $|1 - x^*(x)| < \frac{\epsilon^2}{2}$, there are elements $y \in S_X$ and $y^* \in S_{X^*}$ such that

$$y^*(y) = 1, \quad \|x - y\| < \epsilon, \quad \text{and} \quad \|y^* - x^*\| < \epsilon + \epsilon^2.$$

A lot of attention has been paid to improve these theorems for linear operators between Banach spaces, and latest important results are introduced in this talk.

References

1. E. Bishop and R.R. Phelps, A proof that every Banach space is subreflexive, Bull. Amer. Math. Soc. 67 (1961) 97-98.
2. B. Bollobás, An extension to the theorem of Bishop and Phelps, Bull. London. Math. Soc. 2 (1970) 181-182.